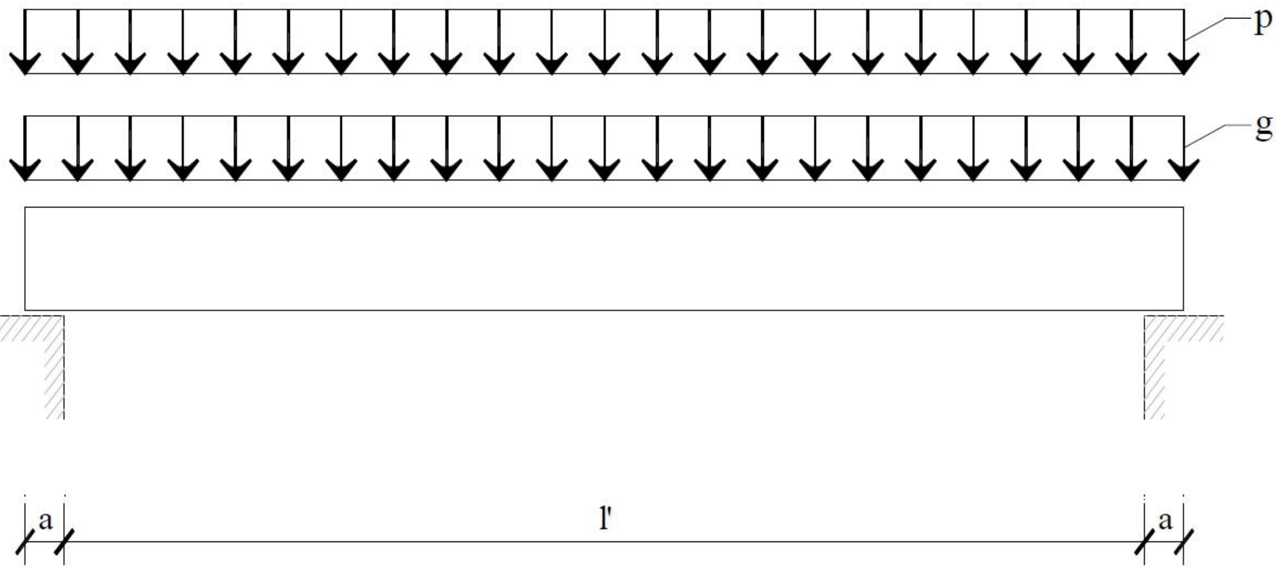
Reinforces Concrete Structures 1.



Tamás Juhász

Reinforces Concrete Structures 1.

Pécs

2019

The Reinforces Concrete Structures 1. course material was developed under the project EFOP 3.4.3-16-2016-00005 "Innovative university in a modern city: open-minded, value-driven and inclusive approach in a 21st century higher education model".

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A Reinforces Concrete Structures 1. tananyag az EFOP-3.4.3-16-2016-00005 azonosító számú,
„Korszerű egyetem a modern városban: Értékközpontúság, nyitottság és befogadó szemlélet egy 21. századi felsőoktatási modellben” című projekt keretében valósul meg.

REINFORCED CONCRETE I

JUHÁSZ TAMÁS

SECTION GEOMETRY:

FREE SPAN: $l' := 4000$

WIDTH OF SUPPORTS: $a_1 := 200$ $a_2 := 200$

WIDTH OF SECTION: $b := 300$

HEIGHT OF SECTION: $h := 500$

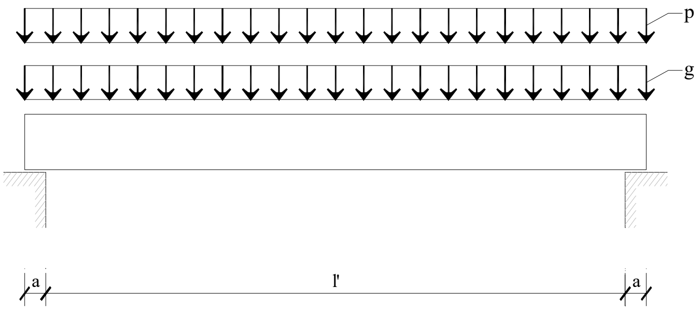
CONCRETE COVER: $c_c := 20$

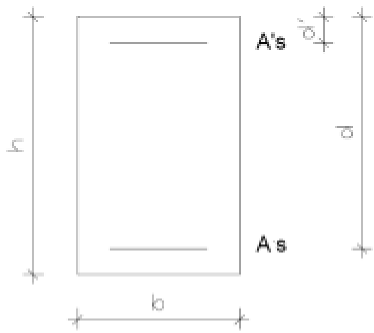
VALUES ARE GIVEN IN mm, N AND THEIR COMBINATIONS

ACCIDENTAL REBAR REPLACEMENT: $\delta := 10$

EFFECTIVE DEPTHS: TENSILE BARS: COMPRESS BARS:

$d := \min(0.9 \cdot h, h - 50) = 450$ $d' := 50$





MATERIAL CLASSES:

C20/25

$f_{ck} := 20$ $\gamma_c := 1.5$ $f_{cd} := \frac{f_{ck}}{\gamma_c} = 13.333$ $f_{ctm} := 2.9$ $\alpha := 1$

$E_{cm} := 33 \cdot 10^3$ $\phi_t := 2.55$ $E_{ceff} := \frac{1.05 \cdot E_{cm}}{1 + \phi_t} = 9760.563$ $\varepsilon_{cu} := 0.35\%$ $f_{bd} := 2.3$

S500B

$f_{yk} := 500$ $\gamma_s := 1.15$ $f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783$ $E_s := 200 \cdot 10^3$

$\xi_{c0} := \frac{560}{700 + f_{yd}} = 0.493$ $\xi'_{c0} := \frac{560}{700 - f_{yd}} = 2.111$

LOADS:

PERMANENT LOADS:

$g_k := 25$ $\qquad \gamma_{g.sup} := 1.35$ $\qquad g_d := \gamma_{g.sup} \cdot g_k = 33.75$

LIVE LOADS, IMPOSED LOADS:

$q_k := 27$ $\qquad \gamma_q := 1.5$ $\qquad \psi_2 := 0.3$ $\qquad q_d := \gamma_q \cdot q_k = 40.5$

LOAD COMBINATION FOR ULS ANALYSIS:

$$p_{Ed} := g_d + q_d = 74.25$$

LOAD COMBINATION FOR SLS ANALYSIS
(QUASI PARMANENT COMBINATION):

$$p_{qp} := g_k + \psi_2 \cdot q_k = 33.1 \qquad \frac{p_{qp}}{p_{Ed}} = 44.579 \cdot \%$$

DESIGN VALUES OF EFFECTS:

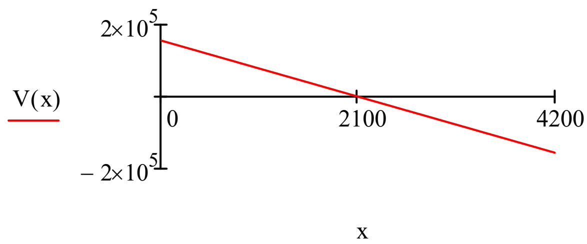
LENGTH OF STATIC MODEL: $l_{eff} := l' + \min\left(\frac{a_1 + a_2}{2}, h\right) = 4200$

LOAD FUNCTION: $L(x) := p_{Ed}$

REACTIONS: $R_A := \frac{1}{2} \cdot \int_0^{l_{eff}} L(x)\,dx$ $\qquad R_A = 1.559 \times 10^5$

SHEAR FUNCTION:

$$V(x) := R_A - \int_0^x p_{Ed}\,dx$$

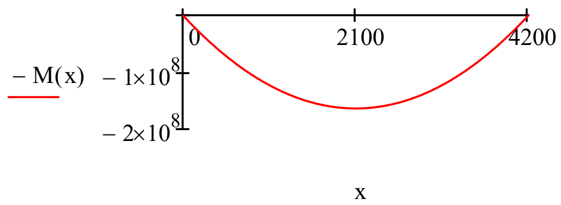


SHEAR (ULS):

$$V_{Ed.max} := V(0) = 1.559 \times 10^5$$

BENDING MOMENT FUNCTION:

$$M(x) := \int_0^x V(x)\,dx$$



BENDING MOMENT (ULS):

$$M_{Ed} := M\left(\frac{l_{eff}}{2}\right) = 1.637 \times 10^8$$

BENDING MOMENT (SLS):

$$M_{qp} := \frac{p_{qp}}{p_{Ed}} \cdot M_{Ed} = 7.299 \times 10^7$$

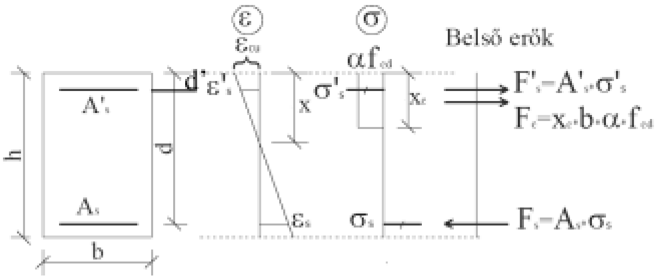
BENDING DESIGN

OPTIMAL BENDING MOMENT:

$$M_0 := \xi_{c0} \cdot d^2 \cdot b \cdot \alpha \cdot f_{cd} \cdot \left(1 - \frac{\xi_{c0}}{2}\right) = 3.011 \times 10^8 \qquad \Delta M := M_{Ed} - M_0 = -1.374 \times 10^8$$

COMPRESS REINFORCEMENT:

$$A'_{s.req} := \begin{cases} \dfrac{\Delta M}{f_{yd} \cdot (d - d')} & \text{if } \Delta M > 0 \\ 0 & \text{otherwise} \end{cases} = 0$$



TENSILE REINFORCEMENT:

$$\sigma'_s := \begin{cases} f_{yd} & \text{if } \dfrac{\xi_{c0} \cdot d}{d'} > \xi'_{c0} \\ 0 & \text{if } A'_{s.req} = 0 \\ E_s \cdot (1.25 \cdot \xi_{c0} \cdot d - d') \dfrac{\varepsilon_{cu}}{1.25 \cdot \xi_{c0} \cdot d} & \text{otherwise} \end{cases} = 0$$

Given

$$x_c := M_{Ed} = x_c \cdot b \cdot \alpha \cdot f_{cd} \cdot \left(d - \frac{x_c}{2}\right) \rightarrow \begin{pmatrix} 797.33179382256389671 \\ 102.66820617743614829 \end{pmatrix}$$

$ORIGIN := 1 \qquad x_{c_2} = 102.668$

$$A_{s.req} := \begin{cases} \dfrac{\xi_{c0} \cdot d \cdot b \cdot \alpha \cdot f_{cd} + A'_{s.req} \cdot \sigma'_s}{f_{yd}} & \text{if } \Delta M > 0 \\ \dfrac{x_{c_2} \cdot b \cdot \alpha \cdot f_{cd}}{f_{yd}} & \text{otherwise} \end{cases} = 944.547$$

APPLIED REINFORCEMENT:

TENSILE:

$\varphi := 16 \qquad n := \text{round}\left(4 \cdot \dfrac{A_{s.req}}{\varphi^2 \cdot \pi} + 0.5, 0\right) = 5 \qquad \zeta := \max(20, \varphi)$

STIRRUP:

$\varphi_k := 8$

COMPRESS:

$$\varphi' := 10 \qquad n' := \begin{cases} \text{round}\left(4 \cdot \dfrac{A'_{s.req}}{\varphi'^2 \cdot \pi} + 0.5, 0\right) & \text{if } A'_{s.req} > 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

$$A'_{s.prov} := n' \cdot \frac{\varphi'^2 \cdot \pi}{4} = 0 \qquad A_{s.prov} := n \cdot \frac{\varphi^2 \cdot \pi}{4} = 1005.31 \qquad \frac{A_{s.prov}}{A_{s.req}} = 106.433 \cdot \%$$

ALLOWABLE NUMBER OF REBARS IN EACH ROWS

$$n_1 := \frac{b - 2(c_c + \varphi_k) + \zeta - \delta}{\varphi + \zeta} = 7.056$$

APPLIED NUMBER OF REBARS:

1ST ROW:

$$n_a := \begin{cases} \mathrm{trunc}(n_1) & \text{if } n_1 < n \\ n & \text{otherwise} \end{cases} = 5$$

2ND ROW:

$$n_f := \begin{cases} (n - n_a) & \text{if } n_1 < n \\ 0 & \text{otherwise} \end{cases} = 0$$

EFFECTIVE DEPTH:

$$d_{prov} := h - c_c - \varphi_k - \frac{\varphi}{2} - \frac{n_f}{n}\cdot(\varphi + \zeta) - \delta = 454$$

$$d'_{prov} := \begin{cases} \left(c_c + \varphi_k + \frac{\varphi'}{2} + \delta\right) & \text{if } A'_{s.prov} > 0 \\ 0 & \text{otherwise} \end{cases}$$

VERIFICATION:

$$x_{c.prov} := \frac{(A_{s.prov} - A'_{s.prov})\cdot f_{yd}}{b\cdot\alpha\cdot f_{cd}} = 109.273$$

$\frac{x_{c.prov}}{d_{prov}} = 0.241$ PLASTIC $\frac{x_{c.prov}}{d'_{prov}} = ▮$ PLASTIC

EC2 REGULATIONS FOR REINFORCEMENT RATIO:

$$\rho_{sl} := \frac{A_{s.prov} + A'_{s.prov}}{b\cdot d_{prov}} = 0.738\cdot\%$$

$$\rho_{sl.min} := \max\left(0.13\%, 0.26\cdot\frac{f_{ctm}}{f_{yk}}\right) = 0.151\cdot\%$$

$\rho_{sl.max} := 4\%$ $\rho_{sl.max} > \rho_{sl} > \rho_{sl.min}$ OK

BENDING CAPACITY:

$$M_{Rd} := x_{c.prov}\cdot b\cdot\alpha\cdot f_{cd}\cdot\left(d - \frac{x_{c.prov}}{2}\right) + A'_{s.prov}\cdot f_{yd}\cdot(d_{prov} - d'_{prov}) = 1.728 \times 10^{8}$$

EFFICIENCY: $\frac{M_{Ed}}{M_{Rd}} = 94.741\cdot\%$

SHEAR DESIGN:

$$V_{Ed}(x) := \begin{cases} V(d_{prov}) & \text{if } x \le d \\ (-V(d_{prov})) & \text{if } x \ge (l_{eff} - d) \\ V(x) & \text{otherwise} \end{cases}$$

$$V_{Ed.red} := V(d) \qquad V_{Ed.red} = 1.225 \times 10^{5}$$

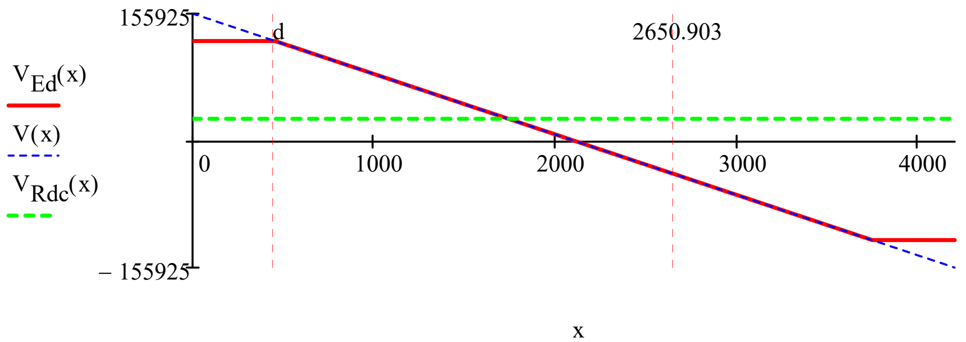
SHEAR RESISTANCE WITHOUT STIRRUPS:

$$\rho_l := \frac{A_{s.prov}}{b \cdot d_{prov}} \qquad k := \min\left(2, \sqrt{\frac{200}{d_{prov}}}\right) \qquad \nu_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot (f_{ck})^{\frac{1}{2}}$$

SHEAR STRENGTH: $$v_{Rdc} := \max\left[\frac{0.18}{\gamma_c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}}, \nu_{min}\right] = 0.195$$

SHEAR RESISTANCE: $$V_{Rdc}(x) := v_{Rdc} \cdot b \cdot d_{prov} \qquad V_{Rdc}(0) = 26611.111$$

SINCE THE EFFECTIVE DEPTH COULD CHANGE ALONG THE BEAM DUE TO THE CHANGE OF LONGITUDINAL REINFORCEMENT, $V_{Rd.c}$ IS NOT CONSTANT.



$$t := \text{root}(V(x) - V_{Rdc}(x), x) = \blacksquare$$

DUE TO THE LINEAR SYMMETRIC SHEAR FUNCTION $V_{Rd.c}$ CUTS THE BEAM TO TWO PARTS. AT THE SUPPORTS $V_{Ed} > V_{Rd.c}$ WHILE ALONG THE MIDSECTION $V_{Ed} < V_{Rd.c.}$ THEREFORE AT THE SUPPORTS THE SHEAR REINFORCEMENT IS DESIGNED WHILE THE MIDSECTION IS REINFORCED WITH THE MINIMUM STIRRUP DENSITY ACCORDING TO THE EC2 REGULATIONS.

$$t_n := l_{eff} - 2 \cdot t = \blacksquare$$

STRUT RESISTANCE:

$$V_{Rd.max} := \frac{1}{2} b \cdot 0.9 \cdot d_{prov} \cdot 0.6 \cdot \left(1 - \frac{f_{ck}}{250}\right) \cdot f_{cd} = 4.511 \times 10^{5}$$

$$\frac{V_{Ed.max}}{V_{Rd.max}} = 34.566 \cdot \%$$

REINFORCEABLE SECTION

REQUIRED STIRRUP DENSITY:

$$A_{sw} := 2 \cdot \frac{\varphi_k^{\ 2} \cdot \pi}{4} = 100.531$$

$$s_{req} := 0.9d \cdot \frac{A_{sw} \cdot f_{yd}}{V_{Ed.red}} = 144.493$$

REEQUIRED REINFORCEMENT RATIO:

$$\rho_w := \frac{A_{sw}}{s_{req} \cdot b} = 0.232 \cdot \%$$

EC2 REGULATIONS:

UPPER BOUND OF SHEAR REINFORCEMENT:

$$\rho_{w.max} := \frac{\frac{1}{2} \cdot 0.6 \cdot \left(1 - \frac{f_{ck}}{250}\right) \cdot f_{cd}}{(1 - \cos(90)) \cdot f_{yd}} = 0.585 \cdot \%$$

LOWER BOUND OF SHEAR REINFORCEMENT :

$$\rho_{w.min} := \frac{0.08 \cdot \sqrt{f_{ck}}}{f_{yk}} = 0.072 \cdot \%$$

THEREFORE:

$$s_{min} := \frac{A_{sw}}{\rho_{w.max} \cdot b} = 57.332$$

$$s_{max1} := \frac{A_{sw}}{\rho_{w.min} \cdot b} = 468.321$$

MAXIMUM ALLOWABLE DISTANCE BETWEEN STIRRUPS:

$$s_{max2} := 0.75 \cdot d_{prov} = 340.5$$

Vagyis:

$$s_{max} := \min(s_{max1}, s_{max2}) = 340.5$$

LOWER BOUND OF STIRRUP DENSITY ALONG THE BEAM (IT IS TO BE APPLIED WHERE $V_{Rdc} > V_{Ed}$) :

$$s_{prov.tn} := 10\,\mathrm{round}\left(\frac{s_{max}}{10} - 0.5, 0\right) = 340$$

WHILE AT SUPPORTS

$$s_{prov.A} := \begin{cases} s_{prov.tn} & \text{if } s_{req} > s_{prov.tn} \\ \left(10 \cdot \mathrm{round}\left(\frac{s_{req}}{10} - 0.5, 0\right)\right) & \text{if } s_{min} < s_{req} < s_{prov.tn} \\ \text{"HIBA"} & \text{otherwise} \end{cases} = 140$$

STIRRUP DENSITY CAN BE REDUCED IN ACCORDANCE WITH THE SHEAR CURVE

LONGITUDINAL REBAR REDUCEMENET, CURTAILMENT:

ANCHORAGE LENGTH (BASIC VALUE):

$$l_{b.d} := \frac{\varphi \cdot f_{yd}}{4 \cdot f_{bd}} = 756.144$$

ANCHORAGE LENGTH (NETTO VALUE):

SMOOTH END: HOOKED END:

$l_{b.net} := l_{b.d}$ $\qquad$ $l_{b.eq.k} := 0.7 \cdot l_{b.net} = 529.301$

MINIMAL ANCHORING:

$$l_{b.min} := \max\left(10 \cdot \varphi, 100, 0.3 \cdot l_{b.d}\right) = 226.843$$

nyomatéki ábra eltolásának mértéke a többlet húzóerő figyelmbevételére:

$$a_l := \frac{1}{2} \cdot 0.9 \cdot d_{prov} = 204.3$$

A húzott mennyiség felének elhagyásának legkorábbi helye:

$$n_{half} := \mathrm{round}\left(\frac{n}{2} + 0.4\right) = 3$$

$$A_{s.half} := n_{half} \cdot \frac{\varphi^2 \cdot \pi}{4} = 603.186$$

$$d_{half} := \left| \begin{array}{l} d_{prov} \ \text{ if } n_f = 0 \\ \left[h - c_c - \varphi_k - \frac{\varphi}{2} - \frac{n_f - n_{half}}{n_a + n_{half}} \cdot (\varphi + \zeta) - \delta \right] \ \text{ if } n_{half} > n_a \end{array} \right. = 454$$

Given

$$x_{c.half} := x_{c.half} \cdot b \cdot \alpha \cdot f_{cd} = A_{s.half} \cdot f_{yd} \ \mathrm{solve}, x_{c.half} \ \rightarrow 65.5636727705695948$$

$$x_{c.half} = 65.564$$

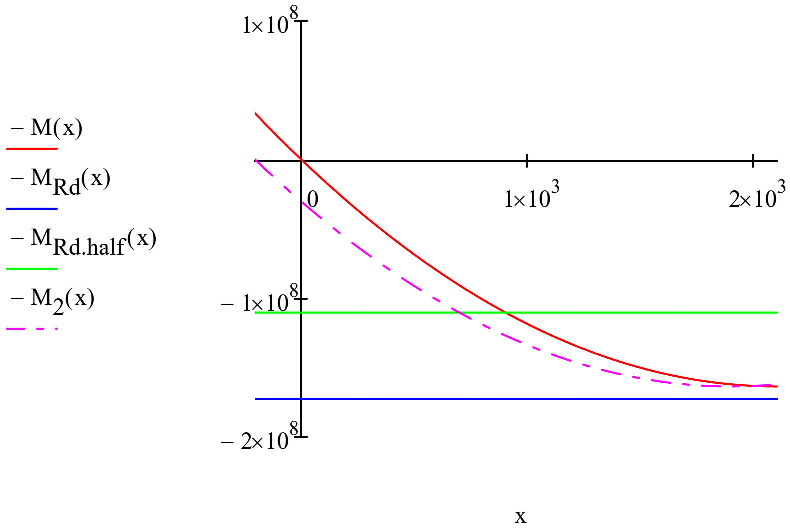
$$M_{Rd.half} := x_{c.half} \cdot b \cdot \alpha \cdot f_{cd} \cdot \left(d_{half} - \frac{x_{c.half}}{2}\right) = 1.105 \times 10^8$$

NUMBER OF CUT BARS:

$n_l := n - n_{half} = 2$ $\qquad$ $M_2(x) := \left(\int_0^{x+a_l} V(x)\,dx\right)$

$M_{Rd}(x) := M_{Rd}$ $\qquad$ $M_{Rd.half}(x) := M_{Rd.half}$

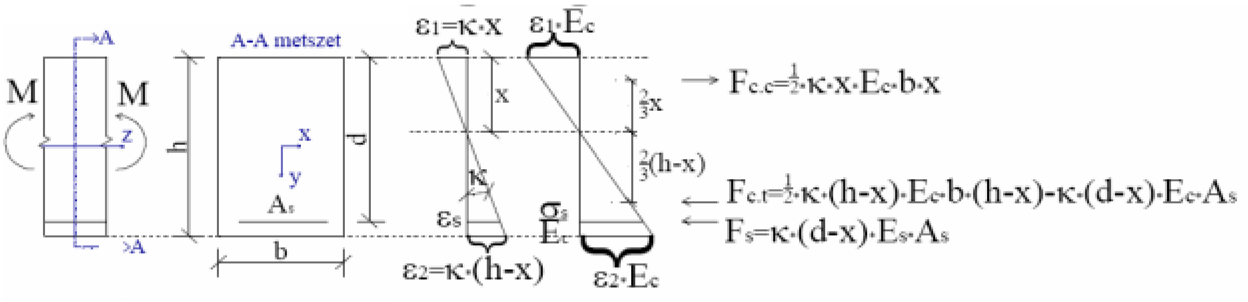
$M_{Ed} = 1.637 \times 10^8$



SERVICEABILITY LIMIT STATE DESIGN:

DEFLECTION:

CROSS-SECTIONAL PROPERTIES (INTACT SECTION):



$$\alpha_E := \frac{E_s}{E_{ceff}} = 20.491 \qquad M_{qp} = 7.299 \times 10^7$$

IDEAL SECTION AREA:

$$A_{iI} := h\cdot b + (\alpha_E - 1)\cdot(A_{s.prov} + A'_{s.prov}) = 1.696 \times 10^5$$

NEUTRAL AXIS POSITION:

$$x_{iI} := \frac{\frac{1}{2}\cdot h^2\cdot b + (\alpha_E - 1)\cdot(A_{s.prov}\cdot d_{prov} + A'_{s.prov}\cdot d'_{prov})}{A_{iI}} = 273.569$$

INERTIA:

$$I_{xiI} := \frac{b\cdot x_{iI}^3}{3} + \frac{b\cdot(h - x_{iI})^3}{3} + (\alpha_E - 1)\cdot\left[A_{s.prov}\cdot(d - x_{iI})^2 + A'_{s.prov}\cdot(x_{iI} - d'_{prov})^2\right]$$

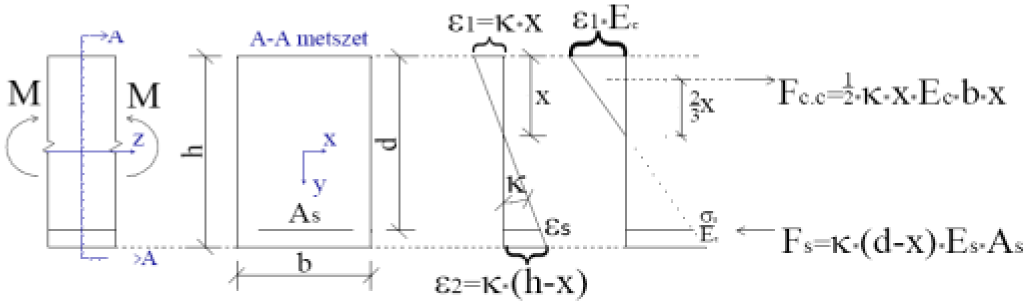
$$I_{xiI} = 3.818 \times 10^9$$

CRACKING MOMENT:

$$M_{cr} := \frac{f_{ctm}\cdot I_{xiI}}{h - x_{iI}} = 4.89 \times 10^7$$

$$\frac{M_{cr}}{M_{qp}} = 67.002\cdot\% \qquad \text{CRACKED SECTION}$$

CROSS-SECTIONAL PROPERTIES (CRACKED SECTION):



Given

$$x_{iII} := x_{iII} = \frac{\frac{1}{2}\cdot x_{iII}^{2}\cdot b + \alpha_E\cdot A_{s.prov}\cdot d_{prov} + (\alpha_E - 1)\cdot A'_{s.prov}\cdot d'_{prov}}{x_{iII}\cdot b + \alpha_E\cdot A_{s.prov} + (\alpha_E - 1)\cdot A'_{s.prov}} \rightarrow \begin{pmatrix} -327.62885369493123555 \\ 190.29939705316096593 \end{pmatrix}*$$

ORIGIN := 1 $\quad x_{iII_2} = 190.299$

$$I_{xiII} := \frac{b\cdot\left(x_{iII_2}\right)^3}{3} + \alpha_E\cdot A_{s.prov}\cdot\left(d - x_{iII_2}\right)^2 + (\alpha_E - 1)A'_{s.prov}\cdot\left(x_{iII_2} - d'_{prov}\right)^2$$

$$I_{xiII} = 2.078 \times 10^{9}$$

STRESS CONTROL:

$$\sigma_c := \frac{M_{qp}}{I_{xiII}}\cdot x_{iII_2} = 6.682 \quad \sigma_s := \alpha_E\cdot\frac{M_{qp}}{I_{xiII}}\left(d_{prov} - x_{iII_2}\right) = 189.741 \quad \sigma'_s := \alpha_E\cdot\frac{M_{qp}}{I_{xiII}}\left(x_{iII_2} - d'_{prov}\right) = 136.926$$

REINFORCEMENT REMAINS ELASTIC

DEFORMATIONS ACCORDING TO SS1 AND SS2:

INTACT SECTION (SS1):

CURVATURE: $\kappa_I := \frac{M_{qp}}{E_{ceff}\cdot I_{xiI}} = 1.958 \times 10^{-6}$ DEFLECTION: $e_I := \frac{5}{384}\cdot\frac{p_{qp}\cdot l_{eff}^{4}}{E_{ceff}\cdot I_{xiI}} = 3.599$

CRACKED SECTION (SS2):

CURVATURE: $\kappa_{II} := \frac{M_{qp}}{E_{ceff}\cdot I_{xiII}} = 3.598 \times 10^{-6}$ DEFLECTION: $e_{II} := \frac{5}{384}\cdot\frac{p_{qp}\cdot l_{eff}^{4}}{E_{ceff}\cdot I_{xiII}} = 6.611$

COMBINATION FACTOR FOR LONG TERM LOADING:

$$\zeta' := 1 - 0.5\left(\frac{M_{cr}}{M_{qp}}\right)^2 = 77.553\cdot\%$$

CALCULATED DEFORMATIONS:

$$e_{EC} := \begin{vmatrix} e_I \ \text{ if } M_{cr} > M_{qp} \\ \left[\zeta'\cdot e_{II} + (1-\zeta')e_I\right] \ \text{ otherwise} \end{vmatrix} = 5.935$$

$$\kappa_{EC} := \begin{vmatrix} \kappa_I \ \text{ if } M_{cr} > M_{qp} \\ \left[\zeta'\cdot \kappa_{II} + (1-\zeta')\cdot\kappa_I\right] \ \text{ otherwise} \end{vmatrix} = 3.23\times 10^{-6}$$

VERIFICATION:

$$e_{max} := \frac{l_{eff}}{200} = 21$$

$e_{max} > e_{EC}$ OK

CRACK WIDTH CONTROL:

REBAR STARIN: $$\varepsilon_s := \frac{\sigma_s}{E_s} = 0.095\cdot\%$$

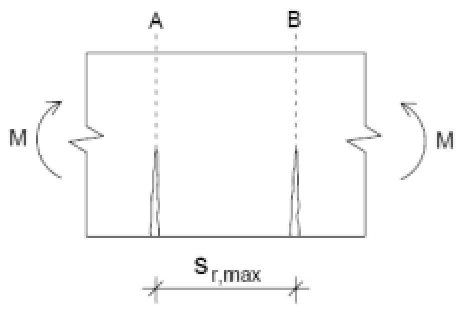
CONCRETE STRAIN BESIDE THE REBARS : $$\varepsilon_c := \frac{f_{ctm}}{E_{ceff}} = 0.03\cdot\%$$

TENSION STIFFENING:
BETWEEN "A" AND "B" THEORETICAL CRACKS THE TENSILED CONCRETE STIFFENS THE REBARS

EFFECTIVE CONCRETE AREA FOR TENSION STIFFENING:

$$h_{c.eff} := \min\left[2.5\left(h - d_{prov}\right), h - \frac{x_{iII}}{3}, \frac{h}{2}\right] \qquad A_{ceff} := b\cdot h_{c.eff} = 34500$$

$$\varepsilon_{sc} := \frac{f_{ctm}\cdot A_{ceff}}{E_s\cdot A_{s.prov}} = 0.05\cdot\%$$



DURABILITY OF LOADS:

LONGTERM LOADING $k_t := 0.4$

STRAIN DIFFERENCE BETWEEN THE CONCRETE AND THE REBARS:

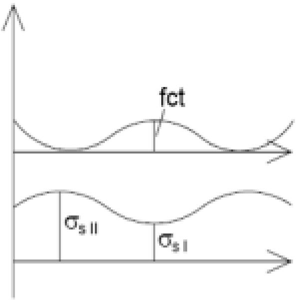
$$\Delta\varepsilon := \varepsilon_s - k_t\cdot\varepsilon_{sc} - k_t\cdot\varepsilon_c = 0.063\cdot\%$$

MAXIMUM RELATIVE DISTANCE BETWEEN TWO CRACKS:

$$s_{r.max} := 3.4\cdot c_c + 0.425\cdot 0.8\cdot 0.5\cdot\frac{\varphi\cdot A_{ceff}}{A_{s.prov}} = 161.344$$

COMPUTED CRACK WIDTH:

$$w_k := s_{r.max} \cdot \Delta\varepsilon = 0.102$$



$w_{k.max} := 0.3$ $\quad w_{k.max} > w_k$ CHECKED

Beton feszültség ellenőrzése tartós terhelésre:

CHARACTERISTIC LOAD COMBINATION: $p_{car} := g_k + q_k = 52$

az ehhez tartozó nyomaték: $M_{car} := \dfrac{p_{car} \cdot l_{eff}^2}{8} = 1.147 \times 10^8$

COMPRESSION STRESS IN CONCRETE: $\sigma_{c.car} := \dfrac{M_{car}}{I_{xiII}} \cdot x_{iII_2} = 10.498$

$\sigma_{c.car.max} := 0.6 \cdot f_{ck} = 12$ ACCEPTABLE